Mehrdad Negahban

CONTINUUM MECHANICS

Fall 2009

WHAT IS CONTINUUM MECHANICS?

- The study of deformation and flow of matter
- The mathematics of continuous systems
- The laws of physics for continuous systems
- The description of problems through differential equations and initial and boundary conditions
- The study of idealizations such as incompressibility and inextensibility
- The study of discontinuities such as shock waves
- …

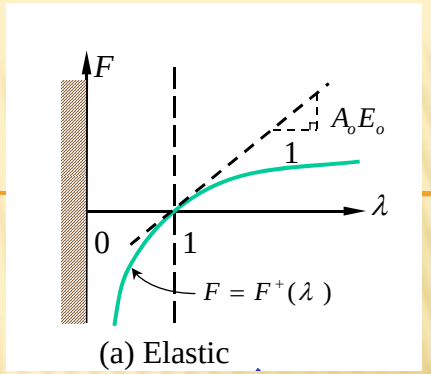
WHAT AREAS ARE WE TRYING TO COVER?

- Linear and nonlinear elasticity
- Viscoelasticity
- Plasticity and viscoplasticity
- Large deformations
- 1-D, 2-D, and 3-D bodies
- Plates and shells
- Waves and jumps
- Fluids and solids
- Failure and fracture
- Thermal and mechanical response …

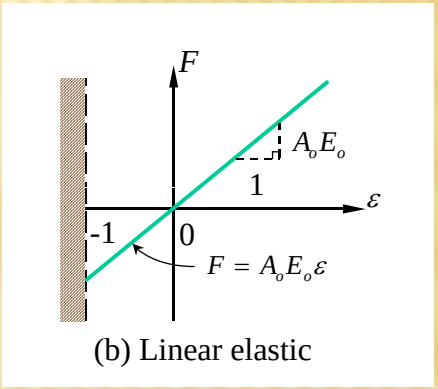
All of first gradient theory!

WHAT WILL WE STUDY?

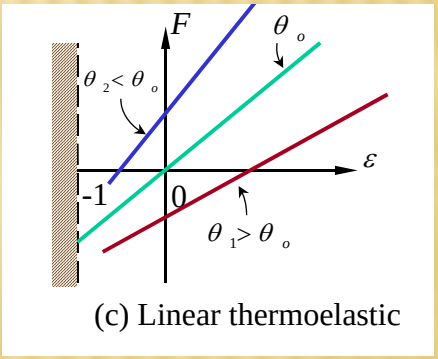
- The 1-D Continuum (bar)
  - Configurations and motion
  - Deformation and strain
  - Integral and differential balance laws
  - Material models
  - Jump conditions



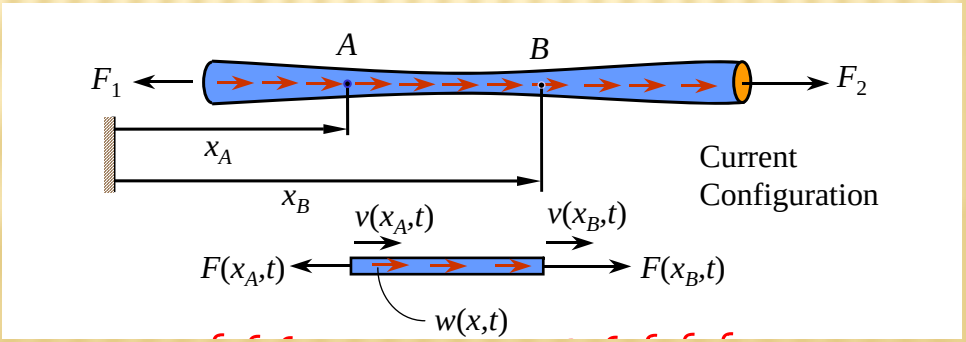
(a) Elastic



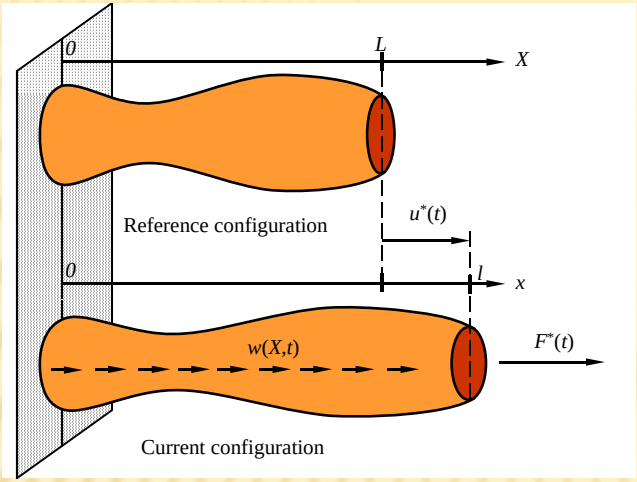
(b) Linear elastic

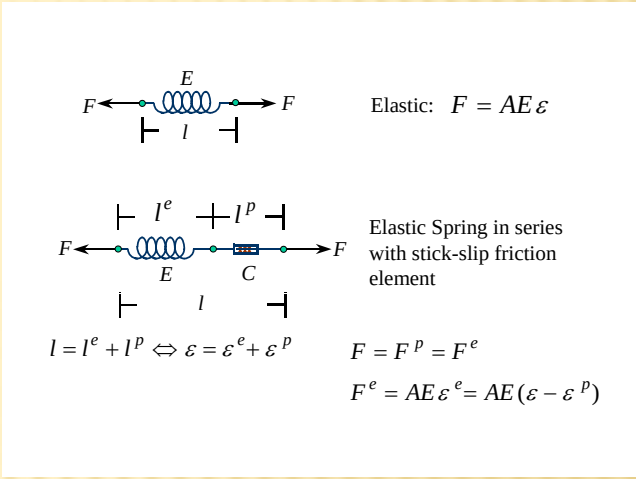


(c) Linear thermoelastic

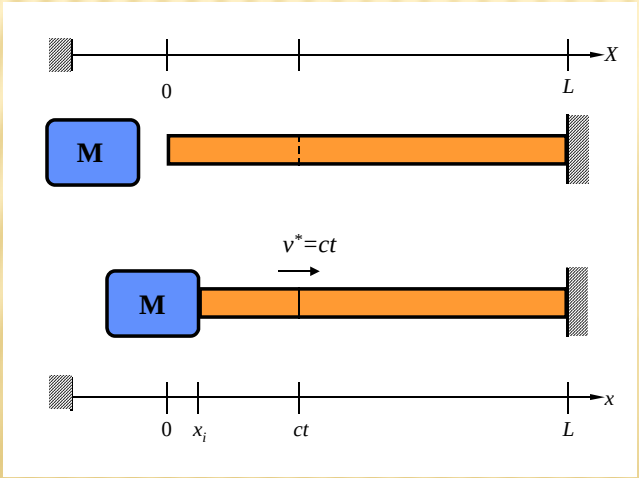


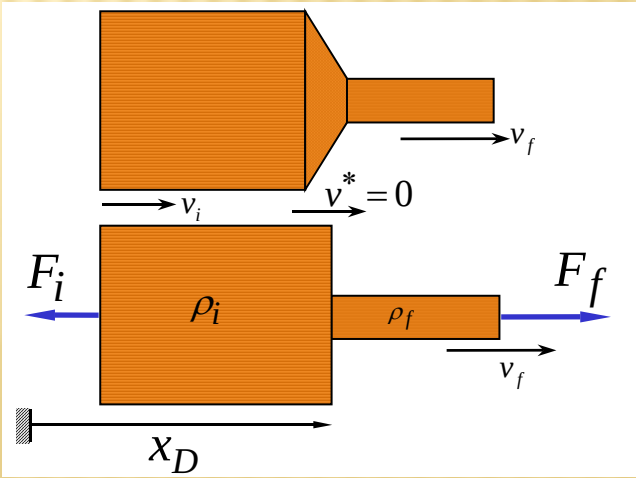
WHAT WILL WE STUDY?





Elastic Spring in series with stick-slip friction element





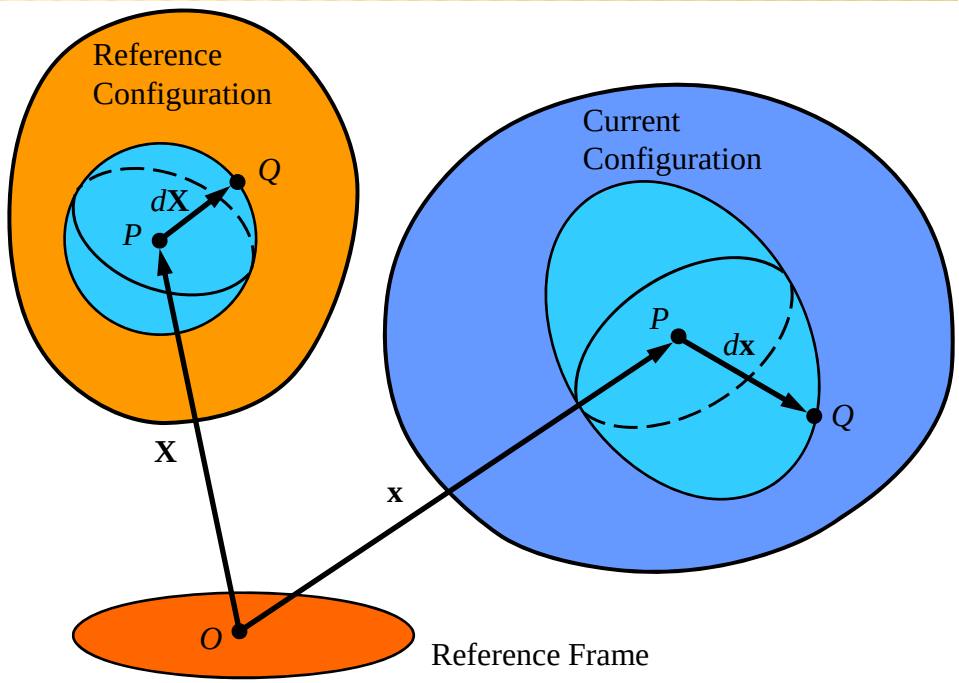
WHAT WILL WE STUDY?

- Index notation
- Vector algebra and calculus
  - Base vectors
  - Vector transformation
  - Calculus
- Tensor algebra and calculus
  - Base tensors
  - Tensor transformations
  - Calculus
  - Identities

A lot of math!

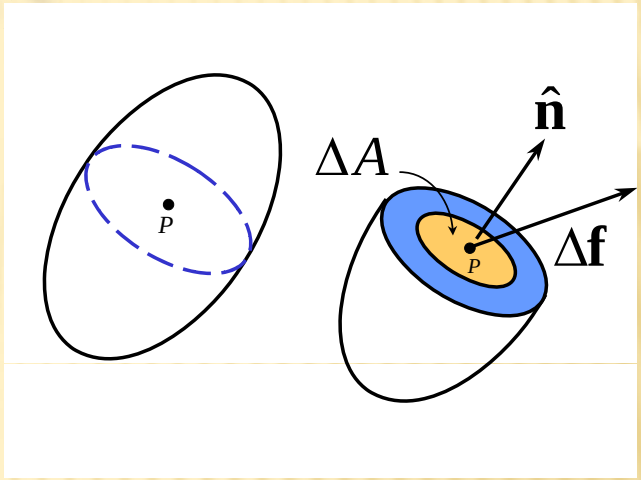
WHAT WILL WE STUDY?

- Kinematics of Deformation
  - Configurations and motion
  - Deformation and displacement gradients $d\mathbf{x} = \mathbf{F} d\mathbf{X}$
  - Velocity gradients
  - Deformation rate
  - Spin tensor
  - Strain tensors

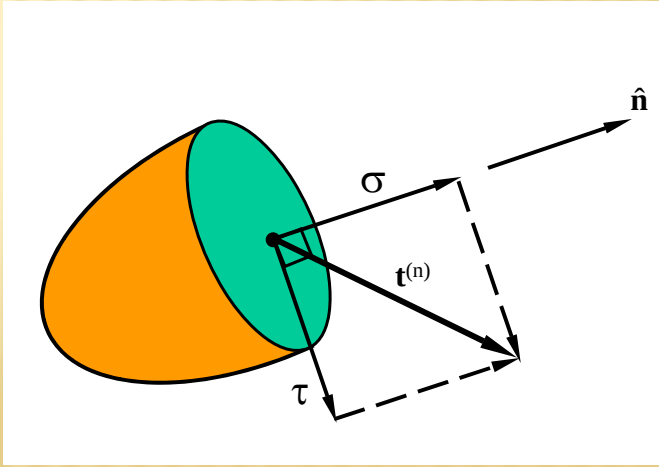


WHAT WILL WE STUDY?

- Load
- Traction
- Stress
  - Cauchy's relation
  - Cauchy stress
  - Nominal stress

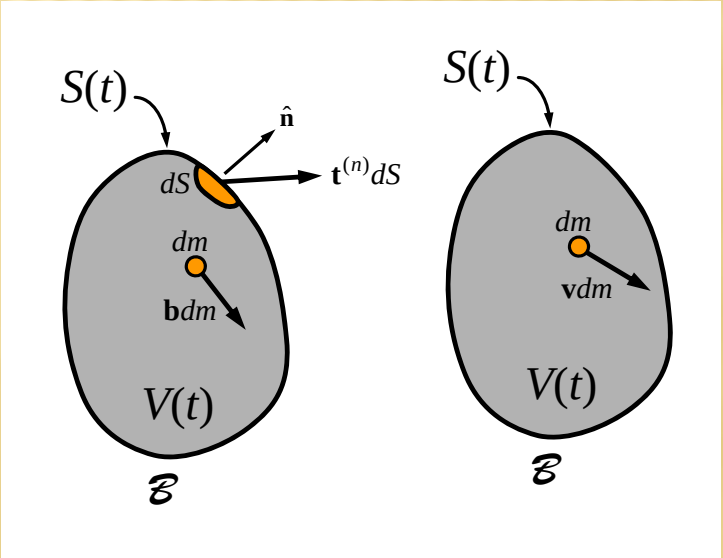


$$\mathbf{t}^{(n)} = \mathbf{T}^T \hat{\mathbf{n}} = \hat{\mathbf{n}}\mathbf{T}$$



WHAT WILL WE STUDY?

- Balance laws
  - Conservation of mass
  - Balance of linear momentum
  - Balance of angular momentum
  - Balance of work and energy
  - Entropy production inequality
  - Jump conditions



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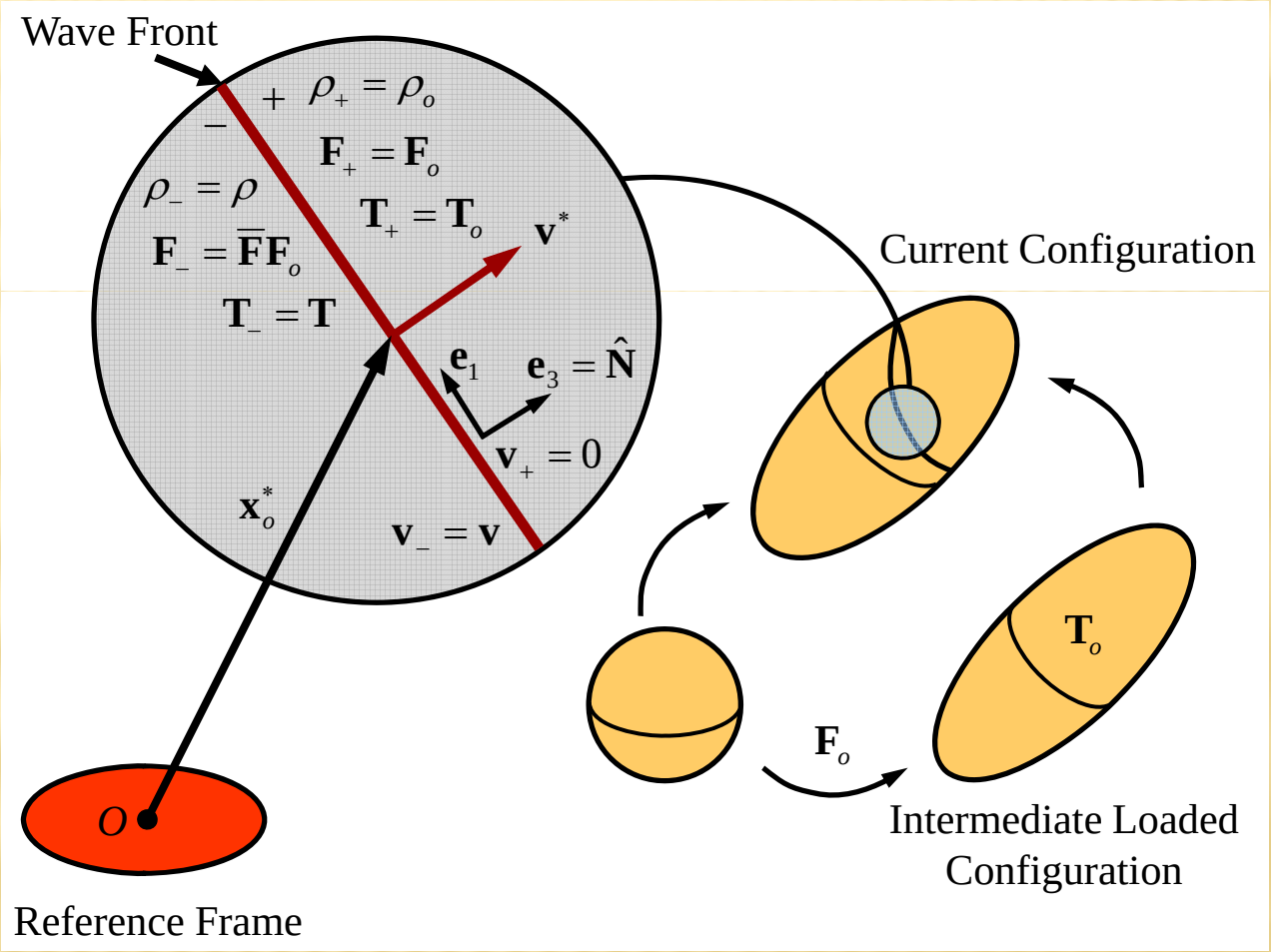
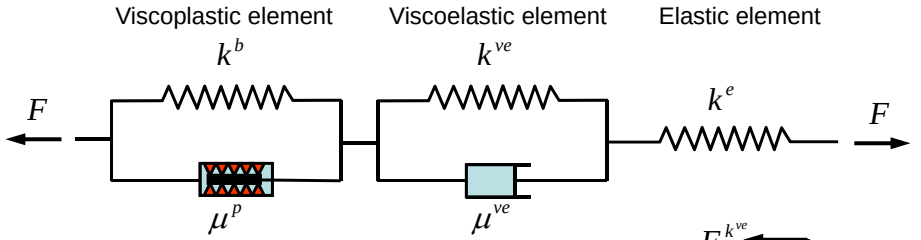


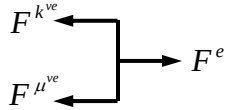
WHAT WILL WE STUDY?

- Theory of constitutive equations
  - First gradient models
  - Rigid body motions
  - Material symmetry
- Elasticity
  - Linear elasticity
  - Internal constraints



$$F = F^e = F^{k^{ve}} + F^{\mu^{ve}} = F^{k^b} + F^{\mu^p}$$

$$\varepsilon = \varepsilon^e + \varepsilon^{ve} + \varepsilon^{vp}$$



Elastic element:

$$F^e = k^e \varepsilon^e$$

Viscoelastic element:

$$F^{k^{ve}} = k^{ve} \varepsilon^{ve}$$

$$F^{\mu^{ve}} = \mu^{ve} \dot{\varepsilon}^{ve}$$

Viscoplastic element:

$$F^{k^b} = k^b \varepsilon^{vp}$$

$$F^{\mu^p} = F - F^{k^b}$$

$$\dot{\varepsilon}^{vp} = \begin{cases} \dfrac{1}{\mu^p} F^{\mu^p} & If \left| F^{\mu^p} \right| > F_y \\ 0 & If \left| F^{\mu^p} \right| \le F_y \end{cases}$$

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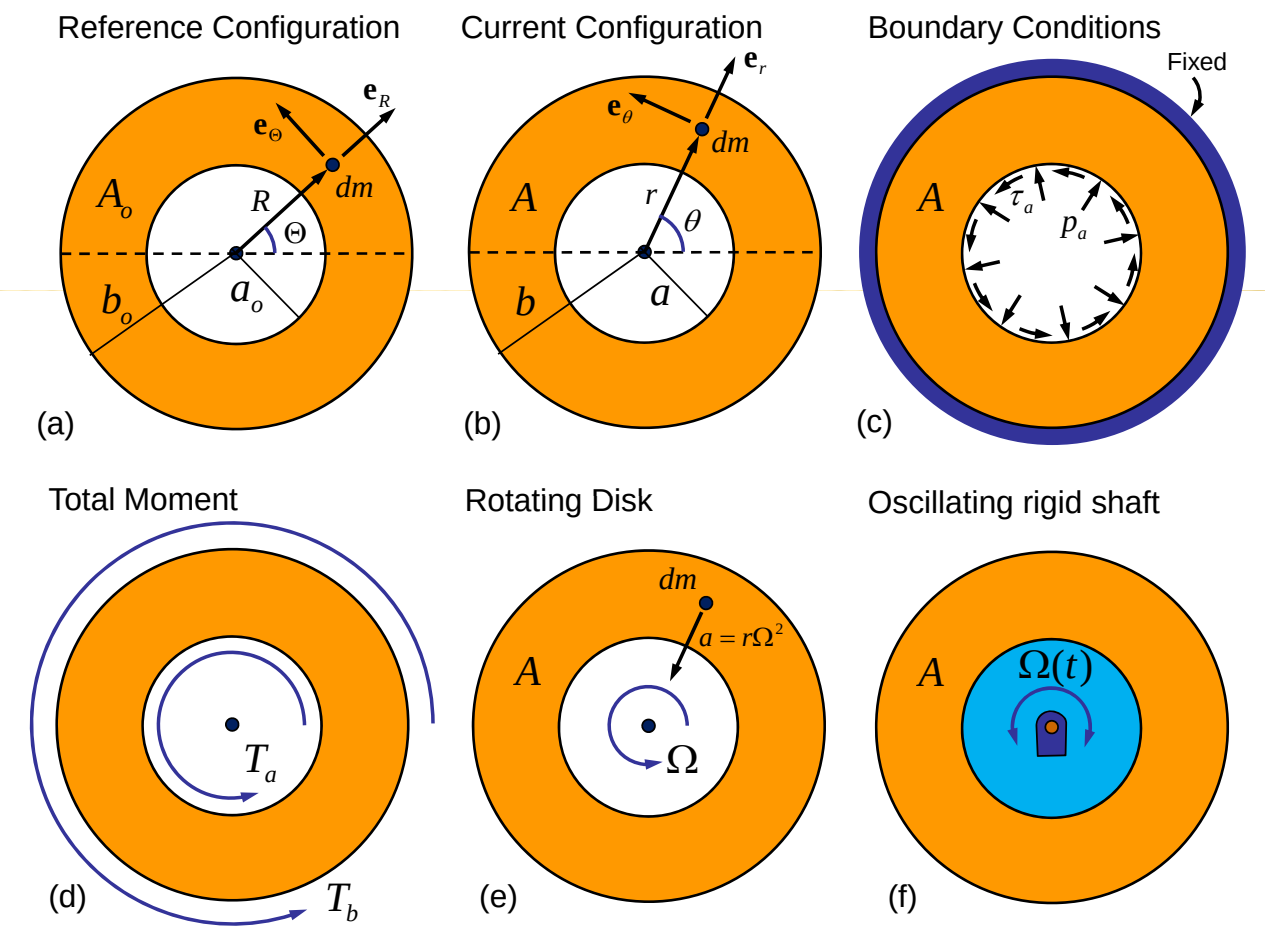

GRADE?

- Class participation: 10%
- Homework: 40%
- Midterm: 20%
- Cumulative final: 30%

- Book: Continuum Mechanics, by A. J. M. Spencer
- Book: Continuum Mechanics, Concise Theory and Problems, by P. Chadwick