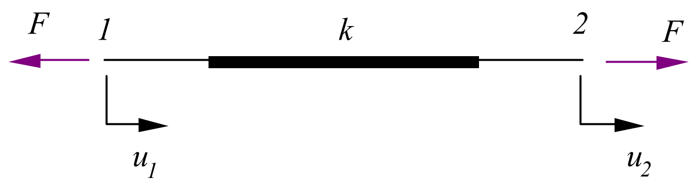
<u>**Stiffness method**</u>

Spring element model:



Load equation: $F = k(u_2 - u_1)$

Load contribution of element spring load on local node 1 and 2:

Take positive load on node as directed to the right:

Load due to spring on local node one:

$$F = k(u_2 - u_1)$$

Load due to spring on local node two:

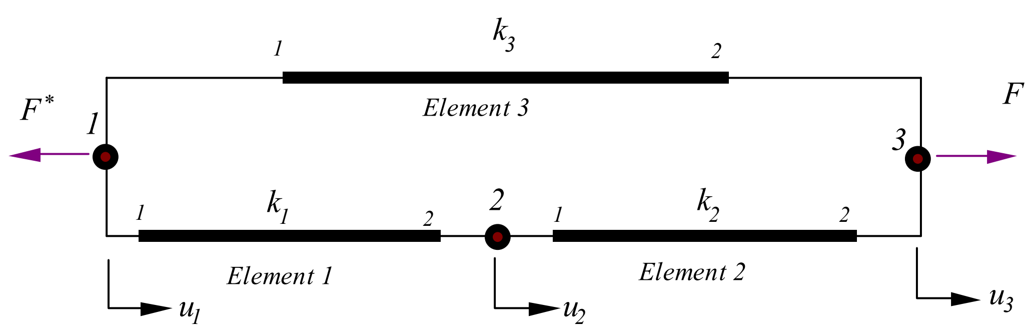
$$-F = -k(u_2 - u_1)$$

Load equations due to spring element e on local nodes put in matrix from:

$$\begin{matrix} & 1 & 2 \\ 1 & -k_e & k_e \\ 2 & k_e & -k_e \end{matrix} \quad \begin{bmatrix} -k_e & k_e \\ k_e & -k_e \end{bmatrix} \begin{bmatrix} u_{e1} \\ u_{e2} \end{bmatrix} = \begin{bmatrix} F_e \\ -F_e \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Local node numbers designated

Example from notes:



Connectivity:

Element #	Global node # For local node 1	Global node # For local node 2
1	1	2
2	2	3
3	1	3

Element equations using global node numbering:

Element 1:

$$\begin{matrix} & 1 & 2 \\ 1 & -k_1 & k_1 \\ 2 & k_1 & -k_1 \end{matrix} \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ -F_1 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Global node numbers designated

Element 2:

$$\begin{matrix} & 2 & 3 \\ 2 & -k_2 & k_2 \\ 3 & k_2 & -k_2 \end{matrix} \quad \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_2 \\ -F_2 \end{bmatrix} \begin{matrix} 2 \\ 3 \end{matrix}$$

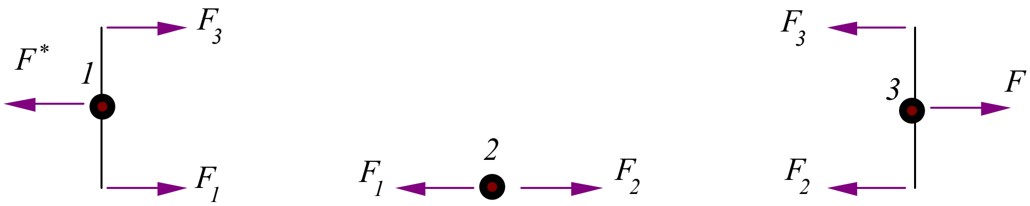
Element 3:

$$\begin{matrix} & 1 & 3 \\ 1 & -k_3 & k_3 \\ 3 & k_3 & -k_3 \end{matrix} \quad \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_3 \\ -F_3 \end{bmatrix} \begin{matrix} 1 \\ 3 \end{matrix}$$

Based on global node number add element stiffness matrix and load vector to global stiffness matrix and load vector:

$$\begin{matrix} & 1 & 2 & 3 \\ 1 & -k_1-k_3 & k_1 & k_3 \\ 2 & k_1 & -k_1-k_2 & k_2 \\ 3 & k_3 & k_2 & -k_2-k_3 \end{matrix} \quad \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_1+F_3 \\ -F_1+F_2 \\ -F_2-F_3 \end{bmatrix}$$

Study load balance at nodes:



$$\begin{cases} F_1 + F_3 = F^* \\ -F_1 + F_2 = 0 \\ F_2 + F_3 = F \end{cases}$$

Substitute these relations into global load matrix:

$$\begin{bmatrix} -k_1 - k_3 & k_1 & k_3 \\ k_1 & -k_1 - k_2 & k_2 \\ k_3 & k_2 & -k_2 - k_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F^* \\ 0 \\ -F \end{bmatrix}$$

Next, impose displacement boundary conditions.